OPTIMAL DESIGN OF REINFORCED CYLINDRICAL SHELLS WITH CORROSIVE WEAR TAKEN INTO ACCOUNT

T. M. Krivoruchko and Yu. M. Pochtman

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Thin-walled structural elements are quite sensitive to corrosion, since even an insignificant diminution in their geometric dimensions due to corrosive wear can result in large stress and strain changes; therefore, they have been utilized more and more in a whole series of engineering areas recently. In this connection, taking account of the influence of corrosion in strength, stability, and longevity analyses, as well as in optimum design, becomes quite important.

An optimal design problem for reinforced cylindrical shells subjected simultaneously to mechanical and chemical destruction is solved in this paper on the basis of experimental dependences of corrosive wear [1] and shell theory equations [2].

Let us consider a thin-walled cylindrical shell of radius r, length L, compressed by an axial load N, hinge-supported at the endfaces and reinforced by stringers and ribs of rectangular cross section. The characteristics of the isotropic shell material are known: the elastic modulus E, the Poisson ratio ν, the density ρ, and the yield point σ_T. The variable parameters are the width and quantity of the stringers and ribs, h_s, k, h_r, k_1; the sheath thickness h and its longevity t (the stringer and rib heights are assumed to be $b_c = \lambda h_c$, $b_r = \lambda h_r$ to eliminate their local buckling [2]).

The constraints assuring shell reliability during its exploitation (without taking account of the influence of the aggressive medium are:

strength

$$(2\pi rh + kh_c b_c)\sigma_T \geqslant N; \tag{1}$$

total buckling

$$(2\pi rh + kh_c b_c)\sigma_{mn} E/(1-\nu^2) \geqslant N, \quad n = 0, \ m = 1, 2, \ldots; \tag{2}$$

local buckling

$$(2\pi rh + kh_c b_c)\sigma_{mn} E/(1-\nu^2) \geqslant N, \ n = 2, 3, \ldots, \ m = 1, 2, \ldots \tag{3}$$

Here m and n are wave-formation parameters in the axial and circumferential directions, respectively; σ_{mn} are critical buckling stresses determined with the discrete nature of the reinforcements taken into account by the method elucidated in [3].

The minimum of the mean rate of construction mass loss during exploitation is taken as target function

$$G = (2\pi rhL + kh_c b_c L + 2k_1 h_r b_r \pi r)\rho/t \to \min. \tag{4}$$

According to [1], the corrosive destruction process has the form

$$dP = f_1 dt + f_2 d\sigma + f_3 dT, \tag{5}$$

where σ is the stress, T is the temperature, P is the corrosive damage parameter, and f_1, f_2, and f_3 are functions describing the process of changes in the geometric and elastic characteristics of the construction as a function of the time, stress, and temperature.

Using the assumptions introduced in [4] about the nature of the chemical destruction of a construction of this class, we obtain two quasistatic systems of equations to determine the critical loads for arbitrary corrosion wear loads given either by analytic expressions such as (5), or by generalization of experimental data:

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$$a_{11s}u + a_{12s}v + a_{13s}w = 0,\; a_{21s}u + a_{22s}v + a_{23s}w = 0,$$
$$a_{31s}u + a_{32s}v + a_{33s}w = 0\; (s = 1, 2). \tag{6}$$

on the basis of an energetic method for a monomial approximation of the displacements [2] within the framework of the Kirchhoff–Love hypotheses. Here s = 1 corresponds to the case of axisymmetric strain, while s = 2 corresponds to skew-symmetric. These two systems are perfectly identical; therefore, all the reasoning performed for one of the systems will remain valid for the other as well, i.e., speaking of system (6), we shall understand either of them.

Taking account of the relationships of the engineering theory of reinforced shells [3] and unconnected thermoelasticity [5], the coefficients in system (6) take the form

$$a_{11s} = a_{11s}(t, \sigma, T) = d_m^2 + \frac{1-\nu}{2} n^2 + 2\gamma_c d_m^2 \sigma_{sn} + 2\mu_r n^2 \sigma_{1m} + 2\lambda_{1r} n^4 \sigma_{1m},$$
$$a_{12s} = a_{21s} = a_{12s}(t, \sigma, T) = a_{21s}(t, \sigma, T) = (-1)^s \frac{(1+\nu)\, d_m n}{2},$$
$$a_{13s} = a_{31s} = a_{13s}(t, \sigma, T) = a_{31s}(t, \sigma, T) = \nu d_m - 2\delta_c d_m^3 \sigma_{sn} -$$
$$- 2\mu_r \left(1 + \frac{h_r}{r}\right) d_m n^2 \sigma_{1m} - 2\lambda_{2r} d_m n^4 \sigma_{1m} - 2\lambda_{1r} d_m n^4 \sigma_{1m},$$
$$a_{22s} = a_{22s}(t, \sigma, T) = \left(n^2 + \frac{1-\nu}{2} d_m^2\right)(1 + a^2) + 2\mu_c d_m^2 \sigma_{s_1 n} +$$
$$+ 2\lambda_{1c}\left(1 - \frac{h_c}{r}\right)^2 d_m^4 \sigma_{s_1 n} + 2\gamma_r \left(1 - \frac{h_c}{r}\right)^2 n^2 \sigma_{2m},$$
$$a_{23s} = a_{32s} = a_{23s}(t, \sigma, T) = a_{32s}(t, \sigma, T) = (-1)^s n \left(1 + a^2 \left(d_m^2 + n^2\right)\right) +$$
$$+ 2\mu_c d_m^2 n \sigma_{s_1 n} + 2\lambda_{1c}\left(1 - \frac{h_c}{r}\right) d_m^4 n \sigma_{s_1 n} + 2\delta_r \left(1 - \frac{h_r}{r}\right) n^3 \sigma_{2m} - 2\gamma_r \left(1 - \frac{h_r}{r}\right) n \sigma_{2m},$$
$$a_{33s} = a_{33s}(t, \sigma, T) = 1 + a^2 \left(d_m^2 + n^2\right)^2 + 2\eta_c d_m^4 \sigma_{sn} + 2\mu_c d_m^2 n^2 \sigma_{s_1 n} +$$
$$+ 2\eta_{r2} n^4 \sigma_{2m} - 4\delta_r n^2 \sigma_{2m} + 2\gamma_r \sigma_{2m} + 2\eta_{r1} n^4 \sigma_{2m} - 4\eta_{r1} n^2 \sigma_{2m} + 2\eta_{r1} \sigma_{2m} +$$
$$+ 2\mu_r \left(1 + \frac{h_r}{r}\right)^2 d_m^2 n^2 \sigma_{1m} + 2\lambda_{3r} d_m^2 n^4 \sigma_{1m} + 4\lambda_{2r} d_m^2 n^2 \sigma_{1m} + 2\lambda_{1r} d_m^2 \sigma_{1m} -$$
$$- \frac{\sigma_x}{E}(1 - \nu^2)\, d_m^2 (1 + 2\gamma_{c0}\sigma_{sn}) - \alpha_T T (1 + \nu)(1 + 2\alpha_c \gamma_{c0} \sigma_{sn})\, d_m^2 -$$
$$- \alpha_T T (1 + \nu)(1 + 2\alpha_r \gamma_{r0} \sigma_{2m})(n^2 - 1).$$

Here σ_x is the magnitude of the axial compressive stresses acting on the shell; $\alpha_c = \alpha_{Tc}/\alpha_T$, $\alpha_r = \alpha_{Tr}/\alpha_T$ (α_T, α_{Tc}, and α_{Tr} are the temperature coefficients of linear expansion of the sheath, the stringer, and the ribs, respectively). The remaining notation is presented in [3].

Taking the initial and boundary conditions into account in addition to (5), the problem of determining the critical axial compressive stresses in this formulation acquires the form

$$a_{11s}u + a_{12s}v + a_{13s}w = 0,\; a_{21s}u + a_{22s}v + a_{23s}w = 0,$$
$$a_{31s}u + a_{32s}v + a_{33s}w = 0,\; dP = f_1 dt + f_2 d\sigma + f_3 dT,$$
$$P(t, \sigma_0, T) = P_\sigma,\; P(t, \sigma, T_0) = P_T,\; P(t_0, \sigma, T) = P_t\; (s = 1, 2), \tag{7}$$

where P_σ, P_T, and P_t are constants obtained experimentally [1]; σ_0, T_0, and t_0 are constants. System (7) can be solved numerically, where the method for its solution is selected as a function of the kind of corrosive destruction. The minimization in the wave-formation parameters realizable in the solution of system (7) affords a possibility of determining the buckling critical stresses.

According to (5) and the assumptions introduced in [4], we now represent the constraints (1)-(3) with the influence of the aggressive medium taken into account, as

$$\frac{2\pi r h (1 + \alpha_1 P) + k h_c b_c}{(1 + \alpha_1 P)^2} \sigma_T \geqslant N,$$

$$\frac{2\pi rh\,(1+\alpha_1 P)+kh_c b_c}{(1+\alpha_1 P)^2}\,\frac{\sigma_{mn}E}{(1+\beta_1 P)\left(1-\nu^2/(1+\beta_2 P)^2\right)}\geqslant N,\quad n=0,\quad m=1,2,\ldots,$$

$$\frac{2\pi rh\,(1+\alpha_1 P)+kh_c b_c}{(1+\alpha_1 P)^2}\,\frac{\sigma_{mn}E}{(1+\beta_1 P)\left(1-\nu^2/(1+\beta_2 P)^2\right)}\geqslant N,$$
$$n=2,3,\ldots,\ m=1,2,\ldots,$$

where $P = P(t, \sigma, T)$; α_1, β_1, and β_2 are coefficients determined from the experimental data [1].

Let us introduce the notation $h = x_1$, $k = x_2$, $k_1 = x_3$, $h_c = x_4$, $h_r = x_5$, $t = x_6$; then the optimization problem formulated is written as follows:

$$G^* = \left(2\pi rLx_1 + \lambda x_2 x_4^2 L + \lambda x_3 x_5^2 \pi r\right)\rho/x_6 \to \min,$$
$$\frac{2\pi r x_1\,(1+\alpha_1 P(x_6))+\lambda x_2 x_4^2}{(1+\alpha_1 P(x_6))^2}\,\sigma_\text{T}\geqslant N,$$
$$\frac{2\pi r x_1\,(1+\alpha_1 P(x_6))+\lambda x_2 x_4^2}{(1+\alpha_1 P(x_6))^2}\,\frac{\sigma_{mn}E}{(1+\beta_1 P(x_6))\left(1-\nu^2/(1+\beta_2 P(x_6))^2\right)}\geqslant$$
$$\geqslant N,\ n=0,\ m=1,2,\ldots,\tag{8}$$
$$\frac{2\pi r x_1\,(1+\alpha_1 P(x_6))+\lambda x_2 x_4^2}{(1+\alpha_1 P(x_6))^2}\,\frac{\sigma_{mn}E}{(1+\beta_1 P(x_6))\left(1-\nu^2/(1+\beta_2 P(x_6))^2\right)}\geqslant$$
$$\geqslant N,\ n=2,3,\ldots,\ m=1,2,\ldots$$

We consider atmospheric corrosion as the aggressive medium. The influence of the stress and temperature are of slight effect on changes in the rate of corrosive wear in atmospheric corrosion, in which connection the corrosive material destruction process is described as $dP/dt = f_1(t)$.

From the condition for the existence of a nontrivial solution of system (6) we obtain an integral equation $\det a_{ij} = 0$ $(i, j = 1, 2, 3)$, which we solve to find the expression to determine the critical axial-compression stresses

$$\sigma_{mn}=\frac{\left(a_{11s}a_{22s}-a_{12s}^2\right)\tilde a_{33s}+2a_{12s}a_{13s}a_{23s}-a_{13s}^2a_{22s}-a_{11s}a_{23s}^2}{d_m^2\,(1+2\gamma_{c0}\sigma_{sn})\left(a_{11s}a_{22s}-a_{12s}^2\right)},\tag{9}$$

where $\tilde a_{33s}=a_{33s}+\dfrac{\sigma_x\,(1-\nu^2)\,(1+2\gamma_{c0}\sigma_{sn})\,d_m^2}{2E}$; $a_{ijs}=a_{ijs}(x_6)$ $(i, j = 1, 2, 3)$.

To be specific, we take the following law of atmospheric corrosion variation [6]:

$$P(x_6) = D(1 + d\exp(-KDx_6))^{-1}.\tag{10}$$

Here D is the maximal value of the degree of destruction, K is a constant that characterizes the reaction to the degree of corrosive destruction at the site under consideration, and d is a corrosion constant.

TABLE 1

N, MN	D, mm	Optimal design parameters						
		x_1, mm	x_2	x_3	x_4, mm	x_5, mm	x_6, yr	G, kg/yr
100	1	4,8	18	6	8,0	7,0	8	7,02
100	2	4,9	18	6	8,2	7,2	8	7,35
100	3	5,0	18	6	8,5	7,3	8	7,68
300	1	6,0	19	8	8,0	7,0	7	9,88
300	2	6,3	19	8	8,2	7,2	7	10,42
300	3	6,5	19	8	8,5	7,3	7	10,84
500	1	7,1	21	9	8,0	7,0	6	13,01
500	2	6,9	21	9	7,8	6,8	6	12,37
500	3	6,6	21	9	7,5	6,5	6	11,44
1000	1	9,2	24	11	8,0	7,0	4	23,63
1000	2	8,8	24	11	7,6	6,6	4	21,45
1000	3	8,5	24	11	7,3	6,3	4	19,83

Solving the nonlinear mathematical programming problem (8) with (9) and (10) taken into account by using the random search method described in [7], we obtain optimal values of the variable parameters.

A numerical experiment is performed for a shell with r = 0.2 m, L = 0.2 m, $E = 6.867 \cdot 10^{10}$ Pa, $\nu = 0.35$, $\lambda = 10$ that is in a corrosive medium with the parameters K = 1649 1/(m·yr), d = 34 for a different loading level and degree of destruction. Optimal design parameters are presented in Table 1; it is seen that the value of the maximal degree of destruction influences neither the longevity of the compressed constant load of the shell nor the number of stringers and ribs but influences the thickness of the sheath and the geometric dimensions of the reinforcing set. As the axial compressive load increases from $1 \cdot 10^8$ to $3 \cdot 10^8$ N, the number of stringers and ribs reinforcing the shell grows, while their geometric dimensions remain constant for each specific value of the maximal degree of destruction. The sheath thickness increases here as the compressive load grows. A uniform diminution in the sheath thickness, the stringer and rib width occurs as the maximal degree of destruction grows when a compressive load $5 \cdot 10^8$ N is achieved; here their quantitities, separately, as well as the longevity of the whole construction, remain constant. This is explained by the fact that, visibly, the growth of mechanical stresses in a metal changes its structure, weakens the adhesion force between its particles resulting in exfoliation of the metal being corroded upon the load reaching a specific value, and in diminution of the shell thickness and the stringer and rib widths as the maximal degree of destruction grows. As should have been expected, the optimal shell longevity is lowered as their loading increases.

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